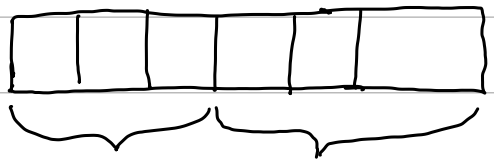
Lecture 19

License Plate Example (Product rule)



letters numbers (0 through 9)

so number of all possible license plates?

$26 \times 26 \times 26 \times 10 \times 10 \times 10$

Sum Rule Example:

There are 5 topics in AI, 7 topics in security, 11 topics in machine learning. You must pick a topic for the final project. How many ways can you finish the final project?

$5 + 7 + 11 = 23$ ways

Example Picking an outfit

- Outfit consists of one shirt and one pant
- you have 5 long sleeve shirts and 4 T-shirts
- you have 3 sweatpants and 5 jeans.
- How many possible outfits are there?
  - ↳ $(5+4) \times (3+5)$

Example We have table with 8 chairs, how many ways can you seat 8 people so that at least one person has new neighbor?

First person to sit: 8 choices

Second person to sit: 7 choices

⋮ ⋮

8th person to sit: 1 choice

So total possibilities is $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8!$

↳ But for every configuration you have 8 other possibilites to seat people that count as the same due to our constraint.

↳ So total possibilites for this problem is $\frac{8!}{8} = 7!$

Example: How many 7 bit binary strings are there that are a palindrome.

Palindrome: If you reverse the string its the same Ex: 1001001 → 1001001

total binary strings (7 bit): 2^7 possibilities

last three bits: 2^3 possibilites

- Use K-to-1 rule
  - ↳ for every 2^3 possibilities only 1 will make the binary string a palindrome.

So # of 7-bit binary strings is $\frac{2^7}{2^3} = \boxed{2^4}$

· Permutation

↳ A permutation refers to an ordered set of objects.

· R-permutation

- Among 100 people you want to give 1st place, 2nd place, 3rd place prize.
- How many ways to do this?

↳ $P(n,r) = \frac{n!}{(n-r)!} = \frac{100!}{(100-3)!} = \frac{100!}{97!} = 100 \times 98 \times 97$

· Interesting permutation questions

- How many permutations of letters ABCDEFGH contain ABC.
  - ↳ treat 'ABC' as one entity
  - ↳ now you have 6 entities
  - ↳ so 6! is the answer

- How many of them contain ABC and FG
  - ↳ Now you have five entities
  - ↳ so 5!

- How many different strings can be formed out of TWITTER
  - 7 letters with 3 T's
  - "k-to-1" rule

$T_0 W I T_1 T_2 E R$ } since there are 3 T's we have 3! ways to
$T_0 W I T_2 T_1 E R$ } express 'TWITTER'

So via k-to-1 rule $\frac{7!}{3!} = 7 \times 6 \times 5 \times 4$